Lecture 1

Thursday, January 21, 2021 12:02 PM

intro + Motivation

Lecture 2

Tuesday, January 26, 2021 12:13 PM

Q — total amount of heat transferred (Joules)

q — heat transfer rate (Watts)

each prime represents a unit of area

q'' — heat flux

$$q = \int q'' \cdot \hat{n} \, dA$$

q' — heat rate per unit length [Watt/m]

$$q' = \frac{dq}{dL} \qquad q = q'L$$

q''' - Heat rate per unit volume

$$q = \int q''' \, dV$$

Usually assume q''' is constant over volume

$$q = q''' V$$

Three Modes of Heat Transfer

Conduction, Convection, and Radiation

Conduction

- transfer of thermal energy through a <u>stationary</u>

substance due to a temperature difference

- arises from microscopic molecular motions

Fourier Law of Heat Conduction

Heat flux in x- direction (1-D)

$$q''_x = -k \frac{dT}{dx}$$

Multidimensional Heat Flux (3-D)

$$\vec{q}'' = -k \nabla T$$

(heat flows from hot to cold

Convection

- transfer of thermal energy by fluid movement

$$\text{Convection} = \underbrace{\text{Conduction}}_{\text{molecular motion}} + \underbrace{\text{Advection}}_{\text{bulk fluid movement}}$$

- 4 classes of convection
  - ① Forced convection
    - fan / pump etc
    - using external power
  - ② Natural (Free) Convection
    - hot fluids rise, cold fluids sink
  - ③ Boili

③ Boiling

④ Condensation

Latent heat transfer by phase change

Newtons law of Cooling

$$q'' = h\,(T - T_\infty)$$

Temp of obj surface

environment temp

h - heat transfer coefficient

Radiation

- heat transfer due to emission / absorbtion of electromagnetic energy by matter

Stefan - Boltzman Law

- Blackbody emissive power

$$E_b = \sigma T^4$$

Stefan Boltzman constant

$5.67 \cdot 10^{-8}\ W/m^2 - K^4$

- Heat emitted per unit area by an ideal surface (Black Body)
    - temp dependant

Radiative Flux: Special Case

$$q'' = \epsilon \sigma \left(T^4 - T_{sur}^4\right)$$

temp of surroundings } - not necessarily same as T_∞

temp of surface

emissivity of surface

$0 \leq \epsilon \leq 1$

All these equations depend on T

Lecture 3

Thursday, January 28, 2021 12:03 PM

What are E, G and q''

E: Emmissive power

- total amount of radiant energy per unit area

$$E = \epsilon E_b$$

E_b → blackbody emmissive power

ϵ emissivity. A measure of how well the surface emits

$$0 \le \epsilon \le 1$$

Irradiation (G)

- total amount of radiant energy per unit area landing on surface

Radiosity

- total amount of radiant energy per unit area

$$J = E + \rho G$$

Flux

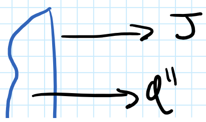
- radiative flux is the balance between J and G

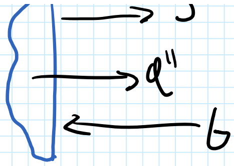
i.e. net energy per unit area leaving the surface

$$q'' = J - G \implies q'' = \epsilon \sigma T^4 - (1-\rho) G$$

$\Downarrow$ assumptions

$$q'' = \epsilon \sigma (T^4 - T_{surr}^4)$$





$q = \epsilon\sigma[T - T_{surr}]$

If it's a blackbody, $\rho = 0 \Rightarrow J = E = \epsilon E_B$

How do we get temperature T

$$\underbrace{\Delta U}_{\text{change in energy as the system undergoes a process}} = \underbrace{Q}_{\text{heat added to system}} - \overbrace{W}^{\text{work done by system}}$$

$$\dot{U} = \dot{E} = q - \dot{W}$$

$$E_{st}^{tot} = \underbrace{KE + PE}_{\text{mechanical energy}} + \underbrace{U}_{\text{internal energy}}$$

$$U = \underbrace{U_t}_{\text{thermal internal}} + \underbrace{U_{chem} + U_{nuc} + U_{elec} + \ldots}_{\text{other forms}}$$

$$\hookrightarrow U_T = U_{T,sensible} + U_{T,latent}$$

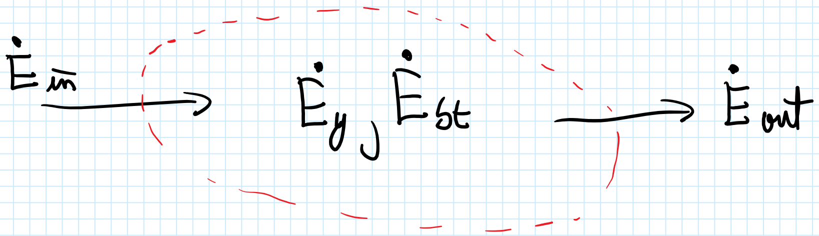
$$E_{st}^{tot} = \underbrace{E_{st}}_{\text{thermal/mechanical energy}} + E_{st,other}$$

$$W^{tot} = W_{mech} + W_{other}$$

$$\dot{E}_{st} = q - \dot{W}_{mech} - \dot{W}_{other} - \dot{E}_{st,other}$$

$$\dot{E}_{st} = \underbrace{q - \dot{W}_{mech}}_{\dot{E}_{in} - \dot{E}_{out}} - \underbrace{\dot{W}_{other} - \dot{E}_{st,other}}_{\dot{E}_g}$$

$$\boxed{\dot{E}_{st} = \underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{energy balance occurring at surface (conduction, convection, radiation etc)}} + \dot{E}_g}$$ essential



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

Energy Balance - Terms

$\dot{E}_{in}$ and $\dot{E}_{out}$: energy flows across surface into/out of the CV

$\dot{E}_g$: thermal generation in CV

$\dot{E}_{st}$: rate of thermal energy stored in CV

↳ related to temperature

$\dot{E}_{in} - \dot{E}_{out} = q - \dot{W}_{mech}$ } Net energy in

Arises from surface phenomena such as

- Radiation
- Convection
- Conduction

} — contribute to q term

Energy Generation

- volumetric

From ① eqn

$$\dot{E}_{st} = \underbrace{\dot{KE} + \dot{PE}}_{\text{neglect this often}} + \dot{U}_t$$

$$\dot{E}_{st} = \rho V c_p \frac{dT}{dt}$$

temperature

density

volume

specific heat

Lecture 4

Tuesday, February 2, 2021 12:05 PM

Energy Balance to Control

- Common assumption $\dot{E}_{st} \approx \dot{U}_t$

$$\underbrace{\rho V c_p \frac{dT}{dt}}_{\dot{E}_{st}} = \underbrace{q - \dot{W}_{mech}}_{\dot{E}_{in}-\dot{E}_{out}} + \overbrace{\dot{E}_g}^{q'''V}$$

net generation rate of thermal + mech energy

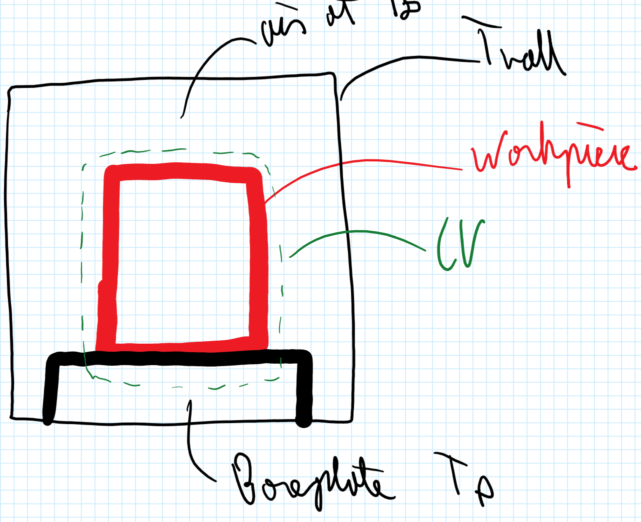
Due to energy conversion from other forms

r.o.c. of Thermal/Mechanical energy stored in object

net inflow rate of thermal/mech energy

Energy Balance Example

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$



$\underbrace{\dot{E}_g = q'''}_{\text{energy generated by induction}} V$, $\dot{E}_{st} = \rho V c_p \frac{dT}{dt}$, identify net inflow $\dot{E}_{in} - \dot{E}_{out}$

$\nearrow 0$

$$\dot{E}_{in} - \dot{E}_{out} = q - \cancel{W}_{mech}$$

↳ conduction to bottom plate

↳ convection 5 sides w/ air

↳ radiation w/ T_{wall}

$$\dot{E}_{in} - \dot{E}_{out} = -q''_{cond,plate} A_B - q''_{conv\,air} A_s - q''_{rad,walls} A_s$$

$$= k_p \frac{dT_p}{dx} - h(T - T_\infty)A_s - \epsilon\sigma(T^4 - T_{wall}^4)A_s$$

$$\rho c_p V \frac{dT}{dt} = k_p \frac{dT_p}{dx} A_B - h(\underset{\substack{\uparrow \\ \text{From}}}{T} - \underset{\substack{\uparrow \\ \text{to}}}{T_\infty})A_s - \epsilon\sigma(\underset{\substack{\uparrow \\ \text{From}}}{T^4} - \underset{\substack{\uparrow \\ \text{To}}}{T_{wall}^4})A_s + q''' V$$

Radiation: Chapter 12

Emission / Absorption

Emission
- transition from higher → lower energy level
- photon is emitted
- propagate outward from material

Absorption

- photon is absorbed
- Transition from lower to higher level

Electromagnetic spectrum

- thermal radiation wavelengths 10^{-7}m - 10^{-4}m

$$\underbrace{\text{UV}}_{\text{part of}} \quad \text{Vis} \quad \text{IR}$$

- Matter is needed to absorb / generate radiation
- cannot have energy w/o atoms / molecules
- Matter is not needed to propagate radiation
  - unlike conduction / convection

- can travel through vacuum
- it can also travel through matter

Lecture 5

Friday, February 5, 2021 11:26 AM

Volumetric heating (heating from within)

- $\dot{E}_g \neq 0$

Radiation

- origin is from energy level transitions
- only mode of heat transfer in vacuum
- important in vacuum and high temp

Two Important Concepts

① Participating vs. Nonparticipating Medium

- medium is stuff between objects

Participating

- can reflect, scatter, absorb
- can also emit radiation

Non-Participating

- radiation passes through w/ no change
- Vacuum, (often assume clean air)

Ex: Earth's Atmosphere

- solar radiation on outside of atmosphere
- not the same as landing on earth's surface

- scattering / absorption

↳ leads to lots of blue light being scattered

– why the sky is blue

- Will usually deal w/ non-participating medium
  (simplifying assumption)

- We will assume Opaque surfaces in this class
  - ultra low transparency ($\tau = 0$)
  - radiation is only absorbed on surface

Radiative Properties

- emissivity (ε)
- Absorptivity (α)
- Reflectivity (ρ)

} often tabulated for different surfaces

- Properties Depend on wavelength
  - example SI photovoltaic absorption
  - perceived color etc

- Spectral Radiation →

- Directional Surface Radiation

How much radiation leaving one object makes it to another?

- depends on medium, intensity and geometry

- Intensity
  - amount of radiant energy in a specific direction
- Surface Emission
  - emits energy in all directions (@ each point)
  - each ray contains an entire spectrum
- Emitted spectral intensity

$$\underbrace{I_{\lambda,e}}_{\text{Intensity}}(\underbrace{x,y}_{\text{location on surface}},\underbrace{\lambda}_{\text{wave length}},\underbrace{\theta,\phi}_{\text{orientation (angle)}},\underbrace{T}_{\text{Temperature}})$$

Assume radiation is uniform across surface

- Doesn't depend on x, y

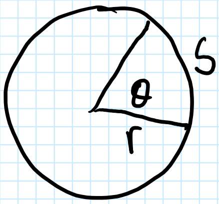
Units $\frac{W}{m^2 \cdot sr \cdot \mu m}$

$m^2 \cdot sr \cdot \mu m$

└ light wavelengths

└ orientation

sr - steradians



$s = r\theta$

$\theta = \frac{s}{r}$ $\quad d\theta = \frac{ds}{r}$

Solid Angle

$$w = A_\perp / R_2 = A\cos\theta / R^2$$

- dealing w/ surface of sphere

$$dw = dA_T / R^2 = dA\cos\theta / R^2$$

Emitted Total Intensity

└ integrated over all wavelengths

$$I_e(\theta,\phi,T) = \underbrace{\int_{\lambda=0}^{\infty} I_{\lambda,e}(\lambda,\theta,\phi,T)\,d\lambda}_{\text{"Adding up intensities of all lights"}}$$

Emissive Power, E

- essentially sum normal component of

- integrate over hemisphere above surface

$$\underbrace{E_\lambda(T)}_{\text{spectral emissive power}} = \int_0^{2\pi}\int_0^{\pi/2} \underbrace{I_{\lambda,e}(\theta,\phi,\lambda,T)\cos\theta}_{\text{normal spectral intensity}}\ \underbrace{\sin\theta\, d\theta\, d\phi}_{\text{solid angle}}$$

Spectrally Tuned "Grow Lights" for plants

$$\underbrace{E(T)}_{\text{total emissive power}} = \int_{\lambda=0}^{\infty} E_\lambda(T)\, d\lambda$$

Need to find spectral intensity

Lecture 6

Tuesday, February 9, 2021 11:59 AM

Intensity

- radiation w/ direction

assume emission is uniform over surface

Emissive Power

- radiation per unit area emitted normal to surface

Why integrate over Hemisphere?

Spectral

- wavelength dependent

Total

- spectral integrated over all wave lengths

Spectral Intensity

$$E_\lambda(T) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\theta,\phi,T)\cos\theta\,\underbrace{\sin\theta\,d\theta\,d\phi}_{d\omega}$$

- we need $I_{\lambda,e}$

Black Body Spectral Intensity

$$I_{\lambda,b} = \frac{2hc_0^2}{\lambda^5\left(\exp(hc_0/\lambda k_B T)-1\right)}$$ } Note, this is a diffuse emitter (doesn't depend on angle)

$$I_{\lambda,e}(\theta,\phi,T) = I_{\lambda,e}(T)$$ } - for Diffuse Emitter

$$E_\lambda(T) = \pi I_{\lambda,e}(T)$$

Blackbody Spectral Emissive Power

$$E_{\lambda,b}(T) = \pi I_{\lambda b}(T)$$

$$= \frac{C_1}{\lambda^5\left(\exp(C_2/\lambda T)-1\right)}$$

$$C_1 = 3.742 \cdot 10^8 \, W\mu m^4/m^2 = 2\pi h c_0^2$$

$$C_2 = 1.439 \cdot 10^4 \, \mu m\text{-}K = hc_0/k_B$$

Plot

- peak for each temp
- as $T \uparrow$, peaks shift to left and increases in magnitude
- huge magnitude increase as $T \uparrow$

$$\underbrace{E_b(T) = \sigma T^4}_{\text{Stefan Boltzmann law}}$$

$$E_b(T) = \pi I_b(T)$$

Wien's Law

$$\lambda_{max} T = C_3 \qquad C_3 = 2897.8\ \mu m\text{-}K$$

- use this to find the dominant color of light that is emitted at a given temperature

Sun (higher temp) peaks in visible region

- lower temp peak in different wavelengths

Radiative Fraction F (Band, range of wavelengths)

- integral of area under curve / total integral

$$\left. F = \frac{\int_0^{\lambda} E_{\lambda b}(T)\, d\lambda}{\sigma T^4} \right\} - \text{Tabulated for } \lambda T \text{ values}$$

Photo diode

- can absorb radiation only in certain range of frequencies

What about $\int_{\lambda_1}^{\lambda_2} E_{\lambda b}$

$$= \left(F_{0-\lambda_2} - F_{0-\lambda_1}\right) \sigma T^4$$

Emissivity

$$q'' = J - G$$

$$= E + \rho G - G = \epsilon E_b + G(1-\rho) = \epsilon E_b - G(\alpha + \tau)$$

ϵ varies w/ material, wavelengths

$$\text{emissivity} = \frac{\text{radiation emitted from real surface}}{\text{radiation emitted from blackbody}}$$

Lecture 7

Thursday, February 11, 2021 12:02 PM

Recap

Wien's Displacement Law

$$\lambda_{max} T = const$$

Band, Radiation Fraction

Diffuse, non-diffuse

- non-diffuse → intensity emitted in different directions is different
- Diffuse → emits same intensity in all directions

Emissivity

- depends on many factors
  - material, surface condition
  - direction, temperature, wavelength

Types of Emissivity

- spectral, directional $\epsilon_{\lambda,\theta}(T) = \dfrac{I_{\lambda,e}(\theta,\phi,T)}{I_{\lambda,b}(T)}$
- total directional $\epsilon_{\theta}(T) = \dfrac{I_{e}(\theta,\phi,T)}{I_{b}(T)}$

- Spectral Hemispherical $\epsilon_\lambda(T) = \dfrac{E_\lambda(T)}{E_{\lambda,b}(T)}$

- Total Hemispherical $\epsilon(T) = \dfrac{E(T)}{E_b(T)}$

Diffuse Assumption

- no angular dependence of radiation
- only deal w/ hemispherical

$$\epsilon = \frac{E}{E_b} = \frac{E}{\sigma T^4}$$

$$E = \int E_\lambda \, d\lambda$$

$$= \int \epsilon_\lambda E_{\lambda,b} \, d\lambda$$

Use Radiation Fraction for piecewise integral

Irradiation

$$q'' = J - G$$

$$= E + \rho G - G = \epsilon E_b - \alpha G$$

Key Difference

- intensity incident

Incident Spectral Intensity

$$I_{\lambda,inc}(\nu,\lambda,\theta,\phi)$$

Spectral Irradiation

$$G_\lambda = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,inc} \cos\theta \sin\theta \, d\theta \, d\phi$$

Total Irradiation

$$G = \int_0^{\infty} G_\lambda \, d\lambda$$

Diffuse Irradiation

- radiation is incident equally from all directions

$$G_\lambda = \pi I_{\lambda,inc} \quad \text{\} in the diffuse case}$$

Defining Absorptivity and Reflectivity

$$\text{absorptivity} = \frac{\text{absorbed radiation}}{\text{incident radiation}}$$

Spectral Dependence of Absorptivity

Different kinds of Reflectivity

$$\text{reflectivity} = \frac{\qquad}{\text{incident}}$$

$$I_{\lambda,\underbrace{e+r}_{\text{emitted + reflected}}} = (\ldots)$$

Use J instead of E

Why should we care about spectral radiative properties

– engineer radiative properties to achieve certain results

Diffuse Gray Surface

↳ emits, absorbs, reflects = in all directions

$\alpha = \epsilon$

Gray – properties are the same at all wavelengths

	Spectral	Total
Emissivity	$\epsilon_\lambda = \frac{E_\lambda}{E_{b,\lambda}}$	$\epsilon = \frac{E}{E_b}$
Absorptivity	$\alpha_\lambda = \frac{G_{abs,\lambda}}{G_\lambda}$	$\alpha = \frac{G_{abs}}{G}$
Reflectivity	$\rho_\lambda = \frac{G_{ref,\lambda}}{G_\lambda}$	$\rho = \frac{G_{ref}}{G}$

Diffuse Grey Surface $\alpha = \epsilon$

- intensity not dependent on wavelength / angle

Key question

- how much radiation leaving one surface makes it to another
    - depends on medium
    - we often assume non-participating medium
    - intensity of radiation leaving

    geometry

    ↳ focus of chapter 13

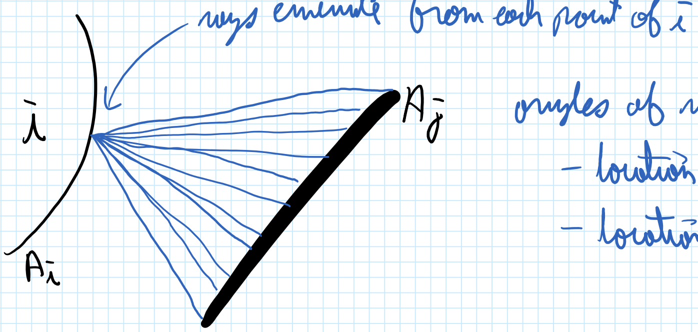
Heat Rate from Surface i to surface j

ie. sun → solar panel

Depends on

- Areas of objects
- Distances

- Angles



angles of ray depend on
- location on surface the originate from
- location on surface they radiate to

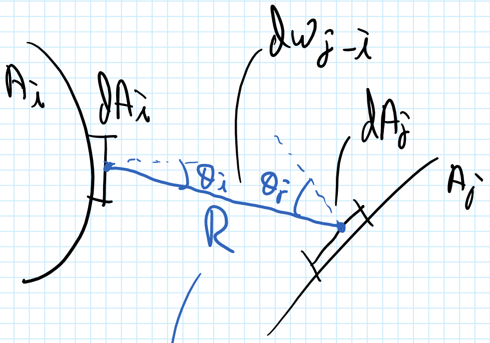
Break Surfaces Into Differential Areas

dA_i and dA_j

Analyze a single ray

Write expression for differential heat ray

Double integration



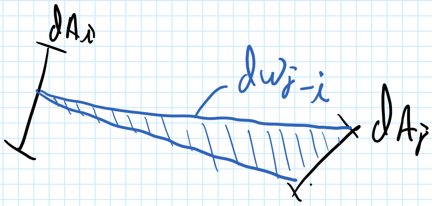
can vary depending on dA_i, dA_j

$$J_i = \int_0^{2\pi}\int_0^{\pi/2} I_{e+r,i}(\theta,\phi)\cos\theta_i \overbrace{\sin\theta\, d\theta_i\, d\phi_i}^{d\omega_i}$$

Total Emitted + Reflected intensity coming from surface i

$$\underbrace{q_{i\to n}}_{W} = \int_{A_i} \underbrace{J_i}_{\frac{W}{m^2}} \underbrace{dA_i}_{m^2} = \int_{A_i} \underbrace{\int_0^{2\pi}\int_0^{\pi/2}}_{\text{over entire hemisphere}} I_{e+r,i}(\theta_i,\phi_i)\cos\theta_i \sin\theta_i\, d\theta_i\, d\phi_i\, dA_i$$

(but how much actually lands on surface j?)



Want to use $d\omega_{j-i}$

$$d\omega_{j-i} = dA_j \cos\theta_j / R^2$$

$$d\omega_{j-i} = dA_{hemi} \cos\theta_{hemi} / r^2$$

$$= \sin\theta_i \, d\theta_i \, d\phi_i$$

$$q_{i\to j} = \int_{A_i}\int_{A_j} I_{e+r,j}(\theta_i,\phi_i) \, \frac{\cos\theta_i \cos\theta_j}{R^2} \, dA_j \, dA_i$$

Integrand

- $I_{e+r,j}$, θ_i, ϕ_i and R depend on specific ray

Range of θ_i, ϕ_i

- size of A_j

Integrating over $dA_i \, dA_j$ pairs

Assume surface is diffuse

$$q_{i\to j} = J_i \iint_{A_i \, A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_j \, dA_i$$

Special case

$A_j, A_k << R^2$ } - rays are essentially ||

$$q_{i,j} \approx J_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} A_j A_i$$

Example

G is coming only from furnace

$$q_{i,j} = J_i \iint_{A_i A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_j \, dA_i$$

$$q_{f,d} = J_f \iint_{A_f A_d} \frac{\cos\theta_f \cos\theta_d}{\pi L^2} \, dA_d \, dA_f$$

$$= G_d A_d$$

Assume $A_d, A_f << L^2$

$$\underbrace{G_d}_{\text{known}} \cancel{A_d} = \underbrace{J_f}_{E_f + \rho \cancel{G_f}^{\,0}} \frac{\cos \overset{0}{\theta_f} \; \cos \overset{0°}{\theta_d}}{\pi \underbrace{L^2}_{\text{solve for this}}} \cancel{A_d} A_f$$

0 because its a hole

Lecture 9

Thursday, February 18, 2021 12:08 PM

Last time

Heat rate leaving A_i landing on A_j

$$q_{i\to j} = J_i \int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2}\, dA_j\, dA_i$$

Special Case, A_j and $A_i \ll R^2$

$$q_{i\to j} \approx J_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} A_i A_j \Big\} - \text{rays are nearly parallel}$$

View Factor (F_{ij})

- Known as configuration or shape factor
- Simple way to account for geometry when calculating radiation exchange between surfaces
- Allows for quick calculations of $q_{i\to j}$
- Physical Meaning
  - fraction of radiation leaving surface i that is intercepted by surface j

$$F_{ij} = \frac{q_{i\to j}}{J_i \cdot A_i} \qquad 0 \leq F_{ij} \leq 1$$

$$F_{ij} = \frac{1}{A_i} \int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2}\, dA_j\, dA_i$$

$$_{ij} \quad \frac{\quad}{A_i}\int_{A_i}\int_{A_j}\frac{\quad}{\pi R^2}$$

Depending on geometry there are tables that characterize the view factors

Can find q_{ij} w/o doing double integral

$$q_{i\to j} = F_{ij} J_i A_i$$

$F_{ij} \neq F_{ji}$ Generally

Reciprocity

$$A_i F_{ij} = A_j F_{ji}$$

Enclosure Rule

$$\sum_j F_{ij} = 1$$

- all energy hitting 1 surface hits some other surface in the enclosure

Self View Factor

$F_{ii} = 0$ Convex

$F_{ii} > 0$ Concave

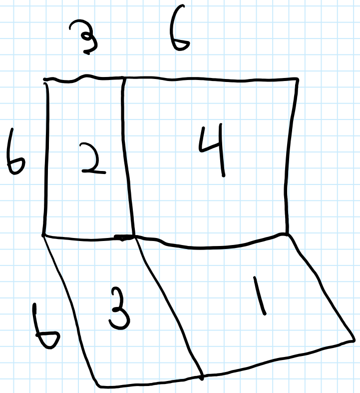
Composite Surface View Factor

$$F \qquad \sum^{n} F$$

$$F_{i(j)} = \sum_{k=1}^{n} F_{ik}$$

$$F_{(j)i} = \frac{A}{A_{(j)}} \sum_{k=1}^{n} F_{ik}$$

Composite Surface Example



Find F_{12}

$$F_{(1+3)(2+4)} = F_{(1+3)}F_2 + F_{(1+3)4}$$

Use reciprocity

$$\frac{A_2 F_{2(1+3)}}{A_{(1+3)}} + \frac{A_4 F_{4(1+3)}}{A_{(1+3)}}$$

$$A_{(1+3)} F_{(1+3)(2+4)} = A_2 F_{2(1+3)} + A_4 F_{4(1+3)}$$

$$= A_2\left(F_{21} + F_{23}\right) + A_4\left(F_{41} + F_{43}\right)$$

By Symmetry $F_{21} = F_{34}$ $A_3 = A_2$

$$A_2 F_{21} = A_2 F_{34} = A_3 F_{34}$$

$$\hookrightarrow A_4 F_{43}$$

$$A_{(1+3)} F_{(1+3)(2+4)} = \underbrace{2A_2 F_{21}}_{2A_1 F_{12}} + A_2 F_{23} + A_4 F_{41}$$

From table

From Table

Lecture 10

Tuesday, February 23, 2021 11:59 AM

View Factor

$$F_{ij} = \frac{q_{i\to j}}{J_i A_i} \Rightarrow q_{i\to j} = \underbrace{F_{ij}}_{} J_i A_i$$

finding view factor?

tables, rules

1. Use tables for 2d-3d geometry
2. Use rules

Reciprocity: $A_i F_{ij} = A_j F_{ji}$

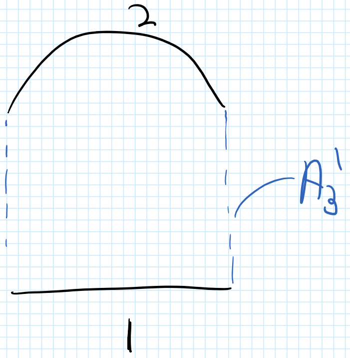
Enclosure Rule $\sum_j F_{ij} = 1$

Self view factors

Composite Surfaces $F_{i(j)} = \sum_{k=1}^{n} F_{ik}$

Hypothetical Surfaces (used to simplify V.F. calculations)

- ex parabolic solar collector



What is F_{13}

$$F_{13} = \underbrace{F_{13l} + F_{13r}}_{= \text{ due to symmetry}} = 2F_{13l}$$

given in a table

Radiative Exchange

- assume isothermal
  - no temp change across surface
- opaque ($\tau = 0$) and diffuse gray ($\epsilon = \alpha$)
- non-participating medium

Radiative Heat Rate

$$q_i'' = J_i - G_i$$

- calculate net heat flow of of i

challenge, finding irradiation

$$q = q''A \qquad q_i = J_iA_i - \underbrace{G_iA_i}_{\text{complicated}}$$

Finding G_iA_i

1. directly compute

$$G_iA_i = \sum_j q_{j\to i} = \sum J_i A_j F_{ji}$$

$$q_i = J_iA_i - G_iA_i = J_iA_i - \sum_j J_jA_iF_{ij}$$

$$= \sum_j \left[J_iA_iF_{ij} - J_jA_iF_{ij}\right]$$

$$q_i = \sum_j \frac{J_i - J_j}{\left(\frac{1}{A_iF_{ij}}\right)} \text{ — } R_{geom,ij}$$

2. Compute from emissive power

$$q_i = J_i A_i + \frac{1}{\rho_i}\left(J_i A_i - \epsilon_i E_{b_i} A_i\right)$$ From Symmetry, $F_{ij} = \frac{1}{3}$

opaque, $\tau = 0$, $\alpha = \epsilon \Rightarrow \rho = 1 - \epsilon$

$q_i =$

Lecture 11

Tuesday, March 2, 2021 12:02 PM

Review

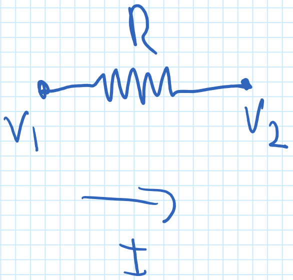
- hypothetical surface
- view factor rules

Eliminate $G_i A_i$

1. directly compute irradiation from all sources
   - leads to q_i in terms of geometrical resistance
2. Substitute radiosity / emissive power expression
   - lead to q_i in terms of surface resistance

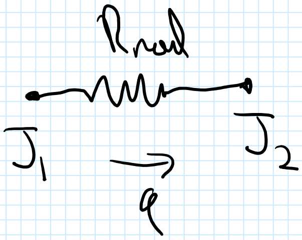
Use radiative resistance concept to study radiation transport

Electrical



$I = (V_1 - V_2)/R$

Thermal Radiation

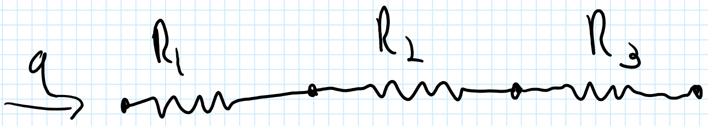


$q = q''A = (J_1 - J_2)/R_{rad}$

Rules for combining Resistors

Series

- same q going through all of them

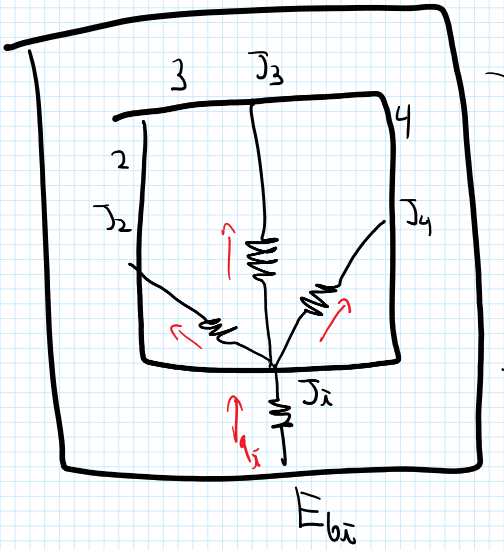


$$R_{eq} = R_1 + R_2 + R_3$$

Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Radiative Energy Balance at Surface



} geometrical resistances $R_{geom\,ij} = \frac{1}{A_i F_{ij}}$

} Surface Resistance $R_{surf,i} = \frac{(1-\epsilon_i)}{\epsilon_i A_i}$

$q_{i\to j}$ heat leaving i landing on j

$q_{j\to i}$ heat leaving j landing on i

$q_{ij} = q_{i\to j} - q_{j\to i}$ Net heat flow from i to j

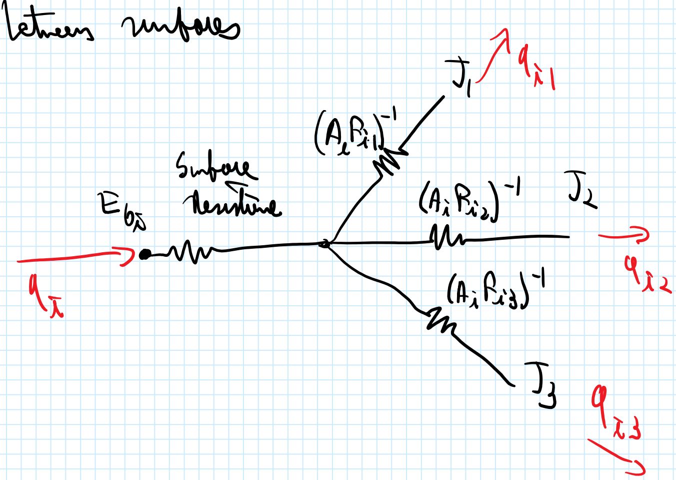
$$q_i = \sum_{j=1}^{N} q_{ij}$$

Surface Radiative Resistance

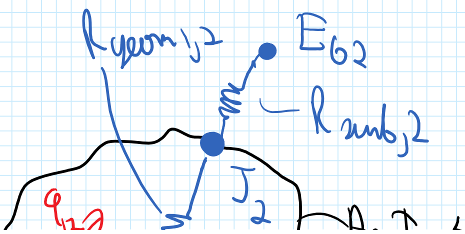
- fictitious node "inside" wall E_{b_i} to surface i

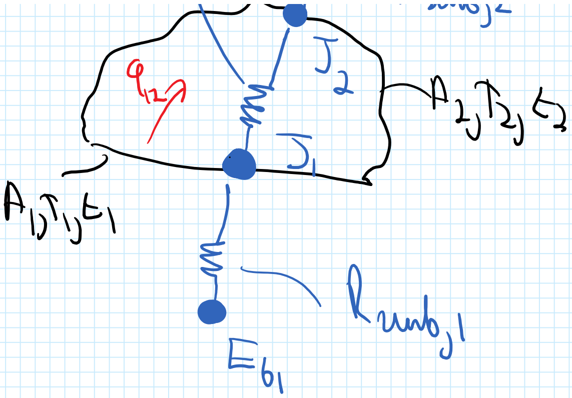
Geometrical Space Resistance

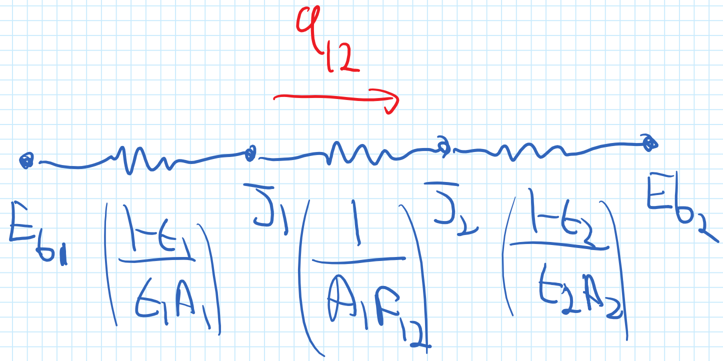
- represents effect of geometry of radiation exchange between surfaces



Two Surface Enclosure





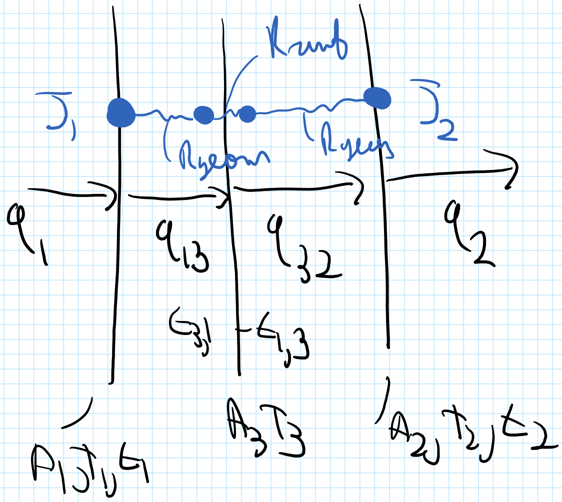


$$q_{12} = (J_1 - J_2)\cdot(A_1F_{12})$$

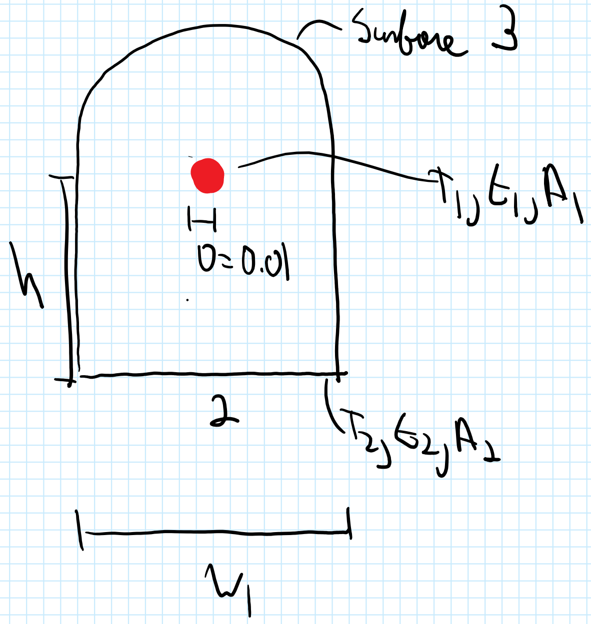
Simplifications

- Black body → $\varepsilon = 1$ ⇒ $E_{b_i} = J_i$

Radiation Shield


Rкинф
J1
J2
Rгеом
Rгеом
q1
q13
q32
q2
ε3,1 -ε1,3
A1,T1,ε1
A3T3
A2,T2,ε2

Furnace Problem



Power per unit length required for S.S. conditions

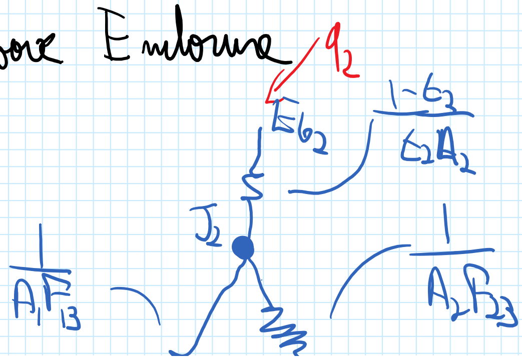
What is temp of furnace wall?

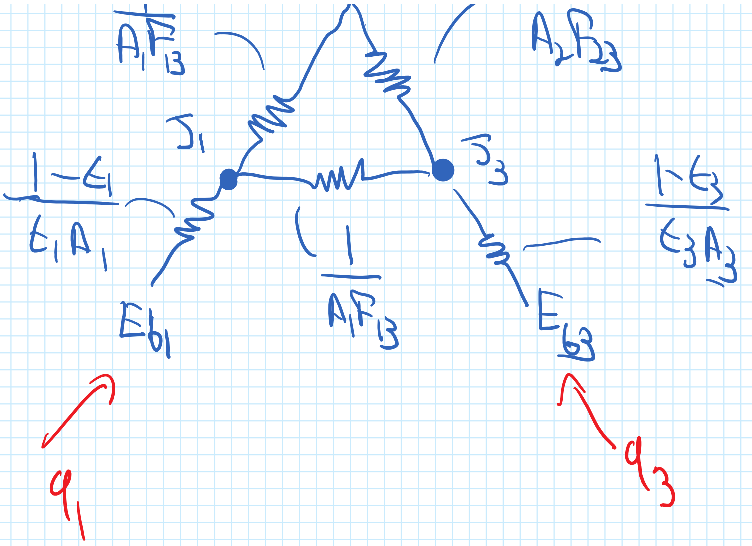
$T_1 = 1500, \epsilon_1 = 1$

$T_2 = 500, \epsilon_2 = 0.6$

$\epsilon_3 = 0.9$ Insulating → $q_3 = 0$

3 Surface Enclosure





$$E_{B1} = \sigma T_1^4, \quad E_{B2} = \sigma T_2^4, \quad E_{B3} = \sigma T_3^4$$

T_1 is known

$$\frac{E_{B1} - J_1}{R_{surf,1}} = \frac{J_1 - J_2}{R_{geom\ 1,2}} + \frac{J_1 - J_3}{R_{geom\ 1,3}}$$

T_2 is known

$$\frac{E_{B2} - J_2}{R_{surf,2}} = \frac{J_2 - J_1}{R_{geom\ 2,1}} + \frac{J_2 - J_3}{R_{geom\ 2,3}}$$

q_3 is known

$$q_3 = \frac{J_3 - J_1}{R_{geom,1,3}} + \frac{J_3 - J_2}{R_{geom\ 2,3}}$$

If we know we have a black body, $\frac{1-\epsilon_i}{\epsilon_i A_i} \Rightarrow 0$

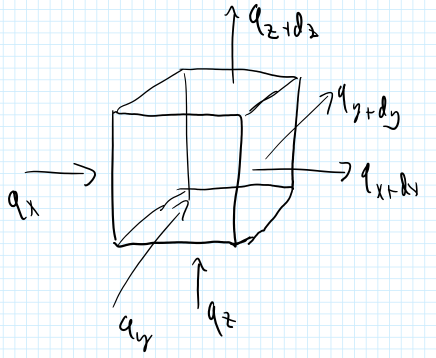
Instead, use $J_1 = \sigma_1 T_1^4$ J_1 is immediately known

Energy Balance on a Differential CV

$dV = dx\,dy\,dz$

- evaluate inflows / outflows / energy storage / energy generation

consider only conductive in flows and out flows



Cartesian Coordinates

$$(In - Out) = (In - Out)_x + (In - Out)_y + (In - Out)_z$$

$$(In - Out)_x = q_x - q_{x+dx}$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}\Big|_x dx + \mathcal{O}^2$$

$$(In - Out)_x = \frac{-\partial q_x}{\partial x}\Big|_x dx$$

$$q_x = q''_x\, dA_x = -k \frac{dT}{dx}\, dy\, dz$$

$$(In - Out)_x = \frac{\partial}{\partial x}\left(k \frac{dT}{dx}\right) dx\, dy\, dz$$

$$(In - Out)_x = k \frac{\partial^2 T}{\partial x^2}\, dx\, dy\, dz$$

can write for y and z directions

$$\boxed{\rho c_p \frac{\partial T}{\partial t} = k \underbrace{\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial T^2}{\partial y^2} + \frac{\partial T^2}{\partial z^2} \right)}_{\nabla^2 T} + q'''}$$

Cylindrical Coordinates?

Spherical Coordinates?

$$(\text{In} - \text{Out})_r = q_r - q_{r+dr}$$

$$(\text{In} - \text{Out})_\theta = q_\theta - q_{\theta+d\theta}$$

$$(\text{In} - \text{Out})_\phi = q_\phi - q_{\phi+d\phi}$$

$$\Rightarrow \quad \rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T + q'''$$

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{q'''}{\rho c_p}, \quad \alpha = \frac{k}{\rho c_p}$$ thermal diffusivity

Del operator in different coordinate systems

Initial and BC

1st order in time, 2nd order in space

1st order in time, 2nd order in space

1 I.C. 2 B.C.

How to write B.C.

In = Out → left = right

Lecture 14

Tuesday, March 16, 2021 12:08 PM

Need initial / boundary conditions

$$q'' = -k \frac{dT}{dx}$$

$$= h \left(T_{from} - T_{to}\right)$$

$$or = \varepsilon\sigma \left[\underset{\text{from}}{T_{sur}^4} - \underset{\text{to}}{T(0,t)^4}\right.$$

One Dimensional Conduction

- heat flows in 1 direction

Linear Heat Flow $T = T(x)$

Radial Heat Flow $T = T(r)$

Steady Conduction

- temp doesn't change w/ time

$$\frac{dT}{dt} = 0$$

Cartesian Example

$$0 = K\frac{d^2T}{dx^2} + q'''$$

Hard/Tedious to Solve Heat equations

If

- no heat generation
- 1-D heat flow
- steady state

Use Conductive Resistances

$$q = \frac{\Delta T}{R} = \frac{T_1 - T_2}{R}$$

$$R_{cond} = \frac{L \text{ — thickness}}{KA \text{ — area}}$$

Table w/ all of Resistances

Critical Insulation thickness

- conductive resistance increases w/ thickness
- convective resistance decreases w/ thickness

Imperfect contact leads to additional thermal resistance

Lecture 15

Thursday, March 18, 2021 12:09 PM

Increase Convection?

- increase surface area
- fan to move air

Fin Heat Transfer Rate

Cannot just use fin area

How do we find q_f? Fin heat transfer rate

$$q_f = -kA_{c,b} \frac{dT}{dx}\bigg|_{x=0}$$

$$\frac{d^2T}{dx^2} + \frac{1}{A_c(x)} \frac{dA_c(x)}{dx} \frac{dT}{dx} = \frac{hP(x)}{kA_c(x)} \left[T(x) - T_\infty\right]$$

Use eta, or efficiency

Generally, applicable to any fin shape

$$\theta = T - T_\infty$$

$$\theta(x) = C_1 \exp(mx) + C_2 \exp(-mx)$$

An easier way, Fin Efficiency

$$\eta_f = \frac{q_f}{q_{f,max}} = \frac{q_f}{h A_f \theta_b}$$

$$q_f = \eta_f \cdot h \cdot A_f \theta_b$$

find this from equations in book/table

$$\eta_f = \frac{\theta_b}{q_f} \Bigg\} - \text{conduction through fin} + \text{convection of surface}$$

$$R_f = \frac{1}{}$$

Fin group

Lecture 16

Tuesday, March 23, 2021 11:56 AM

Usually We Have an array of fins
- not just 1

N - # of Fins

A_b - Total area of exposed base

A_f - Surface Area of single Fin

$$A_t = NA_f + A_b$$

$$R_0 = \frac{1}{\eta_0 h A_t}$$

$$\frac{1}{R_0} = \frac{N}{R_f} + \frac{1}{R_b} = N\eta_f hA_f + hA_b$$

$$\eta_0 = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

$$q_0 = \eta_0 h A_t \theta_b = \frac{\theta_b}{R_0}$$

Chapter 5 - Transient Conduction

Steady state no longer valid

$$\dot{E}_{st} \neq 0$$

Lumped Capacitance Method (Easier)

Direct Solution of Unsteady Heat Equations (Much Harder)

Stored energy in volume V

Lumped Capacitance + Convection at Boundary

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\left(\frac{hA_s}{\rho V c}\right)t\right) = \exp\left(-\frac{t}{\tau}\right)$$

$$\tau = R_{conv} C = \frac{1}{hA_s} \rho V c$$

Lecture 17

Thursday, March 25, 2021 12:03 PM

When can we use lumped capacitance model

- temperature should be nearly uniform throughout object

$$T_{mid} - T_{surf} \ll T_{surf} - T_\infty$$

Biot Number

$$Bi = \frac{hL}{K}$$

L_c = characteristic length

K from solid

Can use lumped capacitance if $Bi \leq 0.1$

L_c is usually center to edge distance

- (distance from largest heat difference)

General L.C

$$\rho V c \frac{dT}{dt} = \underbrace{\dot{E}_{in} - \dot{E}_{out}}_{} + \dot{E}_g$$

↳ simplifies to just convection

What if Biot # > 0.1

solve transient heat equation directly

- spatial and time dimensions

Conduction Shape Factor

↳ analytical solution to heat equation

$$q = Sk(T_1 - T_2)$$

Lecture 18

Thursday, April 1, 2021 12:14 PM

Example problem

- lumped capacitance 4.20

$$q = SK(T_1 - T_2) = \frac{T_1 - T_2}{R}$$

$$R = \frac{1}{SK}$$

heat flows through bores/corners/edges

R_{edge}, R_{corner}, R_{wall}

$$\frac{1}{R_{tot}} = 12\frac{1}{R_{edge}} + 8\frac{1}{R_{corner}} + 6\frac{1}{R_{wall}}$$

Finite element method

Convection:

- goal is to find h

Flow (No slip at surface)

Advection

- energy transfer due to bulk fluid movement

Convective heat flux $\quad h(T_s - T_\infty)$

Conductive heat flux $\quad -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$

$$h(T_s - T_\infty) = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$h = \quad k \frac{\partial T}{}\Big|$$

$$\frac{\partial y|_{y=0}}{T_s - T_\infty}$$

$$q_{adv,x} = \rho e \cdot u\, dy\, dz$$

Lecture 19

Tuesday, April 6, 2021 12:01 PM

Network problem w/ Shape Factor

$$q_{adv} = \underbrace{(\rho e)}_{\text{energy per unit mass}} \underbrace{u}_{x\text{-velocity}} \overbrace{dy\,dz}^{\text{area flowing through}}$$

Full energy equation is valid at any point

How to get T in the thermal boundary layer?

- B.L. layer is so thin we can make simplifications

$$\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}, \quad \frac{\partial^2}{\partial x^2} \ll \frac{\partial^2}{\partial y^2}$$

gradients in y direction are much larger than gradients in x direction

$u \gg v$

Apply B.L. Assumptions

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

$$u \frac{}{\partial x} + v \frac{}{\partial y} = \alpha \frac{}{\partial y^2} + \underbrace{\rho C_p \left(\partial y \right)}_{\text{only important for high speed flows}}$$

$$\alpha = \frac{K}{\rho C_p}$$ } thermal diffusivity constant

What is its actual numerical value

Depends on T, u, and v

Recall continuity, x-momentum etc

h V.S. x, can make an estimate

Approx $\left.\frac{\partial T}{\partial y}\right|_{y=0} \approx \frac{T(\delta)-T(0)}{\delta - 0}$

$\Rightarrow \quad h \approx K/\delta$

increases w/ x

K of fluid

decreases w/ x

How to find h w/o solving for T?

- try to perform experiments

Non-dimensionalize equations

- convert B.L. eq to dimensionless form

Introduce dimensionless variables

x^*, u^*, T^*, p^* etc.

Slight differences

Key dimensionless groups

$\frac{1}{Re}$ Reynolds #

$\frac{1}{Pr\,Re}$

New dimensionless variable

Prandtl Number (Pr)

$Pr = \frac{\nu}{\alpha}$ $\alpha = \frac{k}{\rho c_p}$

- ratio of momentum of diffusivity

Non-dimensionalize h

Nusselt Number

$$Nu = \frac{hL}{k}$$

dimensionless heat transfer coefficient

Biot # Vs Nusselt #

what is difference

$$Bi = \frac{hL}{K_s}, \quad Nu = \frac{hL}{K_f}$$

What variables do T depend on?

Free stream Pressure Gradient

- shape of object determines local pressure gradients

$$T^* = T^*\left(x^*, y^*, \frac{\partial p_\infty^*}{\partial x^*}, Re_L, Pr\right)$$

Need to run experiments @

- different Reynolds #'s

- different shaped objects
- different Pr #'s
- x^a if we care about local Nu #

We can also find avg properties

$$\overline{Nu} = \frac{\bar{h} L}{k} \qquad \bar{h} = \frac{1}{A_s} \int h \, dA_s$$

Local Vs. Avg Nusselt and Sh. #

↳ avg remove positional Dependance

Boundary Layer Equation for Energy

- simplifying full convective energy balance equation

Non-dimensionalizing → 3 key dimensionless groups

Reynolds #

Prandtl #

Nusselt #

$$Pr = \frac{\nu}{\alpha}$$

$$\underbrace{Nu = \frac{hL}{K}}_{\text{can use } Nu, L, K \text{ to find } h} = \frac{\partial T^*}{\partial y^*}\bigg|_{y=0}$$

Functional Dependency

- look for average Nu

$$\overline{Nu} = \frac{\overline{h}L}{K}$$

$$\overline{Nu} = \overline{Nu}\left(Re, Pr, \underbrace{\frac{dP^*_\infty}{dx^*}}_{\text{determined by shape of the object}}\right)$$

Example

$$q_2 = \underbrace{\bar{h}}_{\text{find this}} A_2 (T_{s2} - T_{\infty,2})$$

$$\overline{Nu}_1 = \overline{Nu}_2 \, ? \qquad \overset{\checkmark}{Re_1 = Re_2}, \quad \overset{\checkmark}{Pr_1 = Pr_2}, \quad \text{Shape } 1 = \text{Shape } 2$$

(geometrically similar

$$\bar{h}_1 L_1 = \bar{h}_2 L_2$$

$$\bar{h}_2 = \bar{h}_1 \frac{L_1}{L_2} \qquad\qquad \frac{A_1}{A_2} = \frac{L_1}{L_2}$$

$$\Rightarrow \bar{h}_1 A_1 = \bar{h}_2 A_2$$

Heat and Mass Transfer Analogy

Chilton-Colburn Analogies

Mass transfer is analogous to heat transfer

Mass Diffusion ⟺ heat conduction } Molecular transport

Mass Convection ⟺ heat convection

C_A: Concentration of Species Kmol/m^3

$C_{A,\infty} < C_{A,s}$

Concentration BL thickness δ_c is y value @ which

$$C_A^* = \frac{C_{A,s} - C_A}{C_{A,s} - C_{A,\infty}} = 0.99$$

N_A: Molar Flux of Species A

n_A: Mass Flux of species

h_m: Mass transfer coefficient

D_{AB}: Mass diffusion coefficient of species A in species B

ρ_A: density of species

$$\rho_A = M_A C_A$$

Lecture 21

Tuesday, April 13, 2021 12:02 PM

Main idea

$\overline{Nu}_1 = \overline{Nu}_2$

iff ① $Re_1 = Re_2$

② $Pr_1 = Pr_2$

③ $Shape_1 = Shape_2$

Mass Transfer

- two species
- A (less dominant)
- B (Medium, majority)

Concentration Boundary Layer

- same governing equations as we do for heat transfer
- equations are same, variables are different
  - molar or mass basis

- how to get h_m?
  - same process as getting h
  - species balance across differential C.V.
  - Apply B.L. approx

- Non dimensionalize

3 key groups

Schmidt Sherwood

Re #, $Sc = \frac{\nu}{D_{AB}}$, $Sh = \frac{h_m L}{D_{AB}}$, $\frac{\partial C_A^*}{\partial y^*}\Big|_{y^*=0}$

Heat and Mass Analogy

$$\Rightarrow \quad \frac{\bar{h}}{\bar{h}_m} = \frac{k}{D_{AB}}\left(\frac{D_{AB}}{\alpha}\right)^n = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{1-n}$$

Assume $n = 1/3$ unless told otherwise

Chilton-Colburn Analogy

- If you know friction coefficient you can get h

All have to do w/ gradients of B.L. at surface

- useful because

Evaporative Cooling

- evaporative flux

$$q_{latent} = i_{fg} h_m A \left(\rho_{A,s} - \rho_{A,\infty}\right)$$

$$q_{total} = q_{sensible} + q_{latent}$$

temperature difference

meet evaporation

Review

Lumped Capacitance

L_c = Characteristic Lengths (max T difference)

$$Bi = \frac{hL_c}{K}$$

Lecture 22

Tuesday, April 20, 2021 11:59 AM

Sweaty Runner

$n_a = n_a'' A$

Person is running

$\Rightarrow$ convective mass flux

$$n_a'' = h_m \Big(\underbrace{\overset{(35°C)}{\rho_{sat\,vap,\,skin}}}_{\text{uniform W.V. above sweat}} \underbrace{- \rho_\infty \,(30°C)}_{\text{free stream}} \Big)$$

$$\rho_{sat,vap} = \frac{1}{v_g(35°C)}$$

(gas

$$n_a = A \bar{h}_m \big(\rho_{sat,vap} (35°C) - \rho\,(30°C) \big)$$

$$n_A = A h_m \left(\rho_{sat,vap}(35°C) - \rho_\infty(30°C)\right)$$

how to get $\bar{h}_m$

heat and mass analogy

$$\frac{\bar{h}}{\bar{h}_m} = \underbrace{\frac{k}{D_{AB}}\left(\frac{D_{AB}}{\alpha}\right)^{n=1/3}}_{c} \qquad \alpha = \frac{k}{\rho c_p}$$

how do we get $\bar{h}$

$$q_{sensible} = \bar{h} A (T_s - T_\infty)$$

$$q_{latent} =$$

$$q_{total} = q_{sensible} + q_{latent}$$

Correlations

- results from laminar flow
- Power Law

Lecture 23

Thursday, April 22, 2021 12:08 PM

Internal Flow

Difference from external flow

Boundary layers eventually grow together

- @ distance $x_{,fd}$

fully developed flow

Each boundary layer has its own length

Before $q'' = h(T_s - T_\infty)$

Now $q'' = h(T_s - T_m)$

mean fluid temperature

Thermally Fully developed

$Nu_D = \frac{hD}{k}$ is constant

For an Isothermal wall w/ circular cross section

derived as $Nu = 3.66$

Calculating Outlet Temperature

- need inlet temperature
- length of pipe
- heat transfer per unit length

Energy Balance

Wall heat in + advection in = advection out

density — energy per unit mass

$$q_{adv,x} = \rho u_m e A_c$$

— cross sectional area

mean fluid velocity

$$dq_{wall} = q'' P dx$$

perimeter

$$\boxed{\frac{dT_m}{dx} = \frac{q'' P}{\dot{m} C_p}}$$

differential eq for fluid temp in a pipe @ position x

Integrate to get $T_m(x)$

Constant Surface Temp?

$$T_m(x) = T_s - (T_s - T_{m,i}) \exp\left(-\frac{P x}{\dot{m} C_p} \bar{h}\right)$$

Heat Transfer Rate

Integrate surface heat flux over total pipe S.A.

$$q = \dot{m} c_p (T_{m,o} - T_{m,i})$$

Lot of Internal Flow Correlations

Weird Shaped Tube, use Hydraulic Diameter

Apply Heat and Mass Analogy

Natural Convection

- convection in absence of bulk fluid motion
- No pump/fan, free stream velocity is 0
- density gradient
    - Temperature difference
- Body force
    - gravity

Unstable ⇒ Circulation ⇒ Advection

Lecture 24

Tuesday, April 27, 2021 12:12 PM

Fully developed flow

Nu_D (Sh_D) and h (h_m) are constant

Energy Balance

→ leads to diff eq for mean fluid temp

Heat transferred to fluid in internal flow

$$q = \dot{m} c_p (T_{m,o} - T_{m,i})$$

Turbulent Flow

$$Re_D > 2300$$

Natural/Free Convection

- Forced convection $h \approx$ 1 order of magnitude larger

- can't neglect radiation anymore

Start w/ Boundary Layer Equations

β = Volumetric thermal expansion coef.

Non-dimensionalization

Grashof number (Gr)

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

Rayleigh Number

$$Ra_L = Gr_L \cdot Pr$$

Use rayleigh # to determine turbulent

$$T_f = \frac{T_s + T_\infty}{2} \Bigg\}$$ film temp, use for evaluating fluid properties

Internal free convection

Lecture 25

Thursday, April 29, 2021 12:02 PM

Boiling

- remove heat from surface

saturation (boiling temp)

$$q'' = h(T_s - T_{sat}) = h\Delta T_e$$

excess temperature

depends on if q'' or T_s is controlled

Lots of factors

Boiling correlations

Film condensation correlations

h_{LJ} bind $\emptyset$