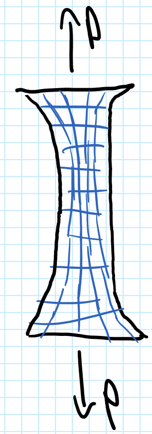
Review Sheet

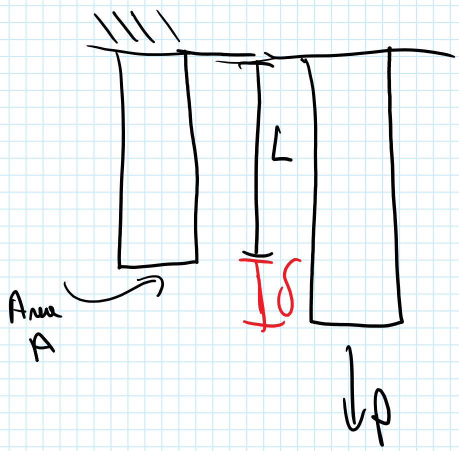
Wednesday, November 6, 2019 1:57 PM

Saint Venant's Principle

- if you go a distance away from the edge, you can use the idealized equations, don't have to worry about edge cases



Axial Loading



$$\sigma = E\epsilon$$

$$\epsilon = \frac{P}{AE}$$

$$\delta = \epsilon \cdot L = \frac{PL}{AE}$$

Statically Indeterminant Problems

- over constrained, too many unknowns to solve w/ just equilibrium equations

Types of Equations

1. Equilibrium
   - sum of the forces/moments
2. Constitutive
   - relate forces/stress to displacement/strain (usually the first)
3. Compatibility
   - relate displacements/strains to other displacements/strains

(i.e. how far can the total system displace?)

Stiffness, K (spring constant)

$$K = \frac{P}{\delta} = \frac{AE}{L}$$

Displacement for non-uniform Material

$$\delta = \int_0^L \frac{P(x)}{A(x)\,E(x)}\,dx$$

Thermal Strain

$$\epsilon_T = \alpha \Delta T$$

α → material property

ΔT → change in temperature

* Only causes stress if system is constrained

$$\epsilon = \frac{\delta}{L} \qquad \underbrace{\delta = \alpha \Delta T \cdot L}_{\text{thermal displacement}}$$

If constrained:

$$\delta_{total} = L\epsilon_{af} + \alpha \Delta T \cdot L = 0$$

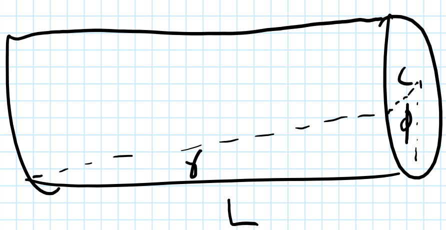
$$\sigma = E\alpha\Delta T$$

For thermal statically indeterminate problems, must include thermal strain in the constitutive equations $\delta = \frac{PL}{AE} + \alpha \Delta T L$

Torsion

- twisting of shaft, moment along axis $\parallel$ to shaft

Shear Strain



$$L \cdot \gamma = \rho \cdot \phi$$

certain radius

Shear strain $\rightarrow$ $\gamma = \frac{\rho \phi}{L}$

angle of twist

Shear Stress

$$\tau = G \cdot \gamma$$

Torque

radius

$$\tau_{max} = \frac{Tc}{J} \qquad \tau = \frac{T\rho}{J}$$

polar moment of inertia

J Solid rod: $\frac{\pi}{2} c^4$

Hollow rod: $\frac{\pi}{2}\left(r_{out}^4 - r_{in}^4\right)$

$$\phi = \frac{TL}{JG} \qquad \phi = \sum_i \frac{T_i L_i}{J_i G_i} \qquad \phi = \int_0^L \frac{T(x)}{J(x)G(x)}\,dx$$

$$\frac{\quad}{JG} \qquad \underbrace{\sum_i \frac{\quad}{J_i G_i}}_{\text{discrete}} \qquad \underbrace{\int_0 \frac{\quad}{J(x)G(x)}\,dx}_{\text{continuous}}$$

$P = T\omega \qquad \omega = 2\pi f$

Power [Watts] $\rightarrow$ $P = T \cdot 2\pi f$

Torque, frequency [Hz]

Stress concentration

- occurs at holes/changes in cross section

$\sigma_{avg} = \dfrac{P}{A} \qquad \sigma_{max} = K \cdot \sigma_{avg}$

constant from table

Statically indeterminant torsion problems